NAG Toolbox for MATLAB

f07pg

1 Purpose

f07pg estimates the condition number of a real symmetric indefinite matrix A, where A has been factorized by f07pd, using packed storage.

2 Syntax

```
[rcond, info] = f07pg(uplo, ap, ipiv, anorm, 'n', n)
```

3 Description

f07pg estimates the condition number (in the 1-norm) of a real symmetric indefinite matrix A:

$$\kappa_1(A) = \|A\|_1 \left\|A^{-1}\right\|_1.$$

Since A is symmetric, $\kappa_1(A) = \kappa_\infty(A) = \|A\|_\infty \left\|A^{-1}\right\|_\infty$.

Because $\kappa_1(A)$ is infinite if A is singular, the function actually returns an estimate of the **reciprocal** of $\kappa_1(A)$.

4 References

Higham N J 1988 FORTRAN codes for estimating the one-norm of a real or complex matrix, with applications to condition estimation *ACM Trans. Math. Software* **14** 381–396

5 Parameters

5.1 Compulsory Input Parameters

1: **uplo – string**

Indicates how A has been factorized.

uplo = 'U'

$A = PUDU^{\mathrm{T}}P^{\mathrm{T}}$, where U is upper triangular.

uplo = 'L'

$A = PLDL^{\mathrm{T}}P^{\mathrm{T}}$, where L is lower triangular.

Constraint: **uplo** = 'U' or 'L'.

2: **ap**$(*)$ **– double array**

Note: the dimension of the array **ap** must be at least $\max(1, \mathbf{n} \times (\mathbf{n}+1)/2)$.

The factorization of A stored in packed form, as returned by f07pd.

3: **ipiv**$(*)$ **– int32 array**

Note: the dimension of the array **ipiv** must be at least $\max(1, \mathbf{n})$.

Details of the interchanges and the block structure of D, as returned by f07pd.

4: **anorm – double scalar**

The 1-norm of the **original** matrix A, which may be computed by calling Missing 'id'. **anorm** must be computed either **before** calling f07pd or else from a copy of the original matrix A.

Constraint: **anorm** ≥ 0.0.

5.2 Optional Input Parameters

1: **n – int32 scalar**

Default: The dimension of the array **ipiv**.

n, the order of the matrix A.

Constraint: **n** ≥ 0.

5.3 Input Parameters Omitted from the MATLAB Interface

work, iwork

5.4 Output Parameters

1: **rcond – double scalar**

An estimate of the reciprocal of the condition number of A. **rcond** is set to zero if exact singularity is detected or the estimate underflows. If **rcond** is less than ***machine precision***, A is singular to working precision.

2: **info – int32 scalar**

info $= 0$ unless the function detects an error (see Section 6).

6 Error Indicators and Warnings

Errors or warnings detected by the function:

info $= -i$

If **info** $= -i$, parameter i had an illegal value on entry. The parameters are numbered as follows:

1: **uplo**, 2: **n**, 3: **ap**, 4: **ipiv**, 5: **anorm**, 6: **rcond**, 7: **work**, 8: **iwork**, 9: **info**.

It is possible that **info** refers to a parameter that is omitted from the MATLAB interface. This usually indicates that an error in one of the other input parameters has caused an incorrect value to be inferred.

7 Accuracy

The computed estimate **rcond** is never less than the true value ρ, and in practice is nearly always less than 10ρ, although examples can be constructed where **rcond** is much larger.

8 Further Comments

A call to f07pg involves solving a number of systems of linear equations of the form $Ax = b$; the number is usually 4 or 5 and never more than 11. Each solution involves approximately $2n^2$ floating-point operations but takes considerably longer than a call to f07pe with one right-hand side, because extra care is taken to avoid overflow when A is approximately singular.

The complex analogues of this function are f07pu for Hermitian matrices and f07qu for symmetric matrices.

9 Example

```
uplo = 'L';
ap = [2.07;
     4.2;
     0.2230413840558341;
     0.6536583767489105;
     1.15;
     0.8115010321439103;
     -0.5959697237786296;
     -2.59067708640519;
     0.3030846795506181;
     0.4073851981348882];
ipiv = [int32(-3);
     int32(-3);
     int32(3);
     int32(4)];
anorm = 11.29;
[rcond, info] = f07pg(uplo, ap, ipiv, anorm)
```

```
rcond =
    0.0132
info =
           0
```